**Title: Long-Term Chaotic Nature of the Planets’ Orbits in The Solar System**

**Abstract:**

This report aims to explore the prevalence of chaos in the Solar System, mainly analysing the chaotic behaviour of the orbits of the inner planets (Mercuy, Venus, Earth and Mars) and outer planets (Jupiter, Saturn, Uranus and Neptune) of the Solar System. We will also look at 2 of the main causes of this chaotic nature in their orbits, the n-body problem and orbital resonance overlaps.

**Introduction:**

The concept of chaos theory can be traced back to the 19th Century from French mathematician Henri Poincaré[[1]](#footnote-12823). While he did not discover chaos theory, his work on the n-body problem laid the foundations of chaos theory, and defined the main characteristic of chaotic systems, that their end results are highly sensitive to the initial conditions of the system[[2]](#footnote-22261). Chaos theory is then later popularized by American mathematician and meteorologist Edward Lorenz who rediscovered chaos when he was trying to simulate weather patterns and realised that any minute differences in the initial conditions of his weather simulations caused completely different predictions of the weather[[3]](#footnote-15882). The term “the butterfly effect” was later created, influenced from a famous sentence Edward Lorenz said in one of his lectures: “a butterfly flapping its wings in Brazil can produce a tornado in Texas” to represent a chaotic system’s sensitivity to the initial conditions and was then popularized through bestselling books[[4]](#footnote-18470).

Ever since Isaac Newton’s discovery of his 3 laws of motion and the law of universal gravitation[[5]](#footnote-28827), and Johannes Kepler’s discovery of his 3 laws of planetary motion[[6]](#footnote-5532), people began to believe that the Solar System is predictable and completely deterministic[[7]](#footnote-28213). However, as technology improved, scientists began to realise that while the Solar System as a whole is predictable in the short run (< million years) (different components of the Solar System has varying durations of predictability before they become unpredictable, but are generally predictable up to a million years), in the long run (> million years), their calculations and prediction models start to fail due to the prevalence of chaos in the Solar System[[8]](#footnote-30620).

**Methodology:**

This report sourced its data from various research papers on the topic of chaos in the Solar System, as well as information from Singapore University of Technology and Design (SUTD)’s SHARP Honours sessions on the topic of Non-linear Dynamics and Chaos.

**Results:**

**Chaotic behaviour caused by the n-body problem:**

The n-body problem is defined as: “Given the quasi-steady orbital properties (instantaneous position, velocity and time) of a group of celestial bodies, predict their interactive forces; and consequently, predict their true orbital motions for all future times.”[[9]](#footnote-23870)

The Solar System is made up of many celestial bodies, including the Sun, the planets, moons, asteroids, etc, all of which are constantly interacting with one another (mainly) gravitationally at varying degrees depending on the size of the celestial bodies, but are still affecting one another nonetheless. Hence, celestial mechanics in the Solar System is effectively an n-body problem. In the context of the orbits of the planets, in the short run, special configurations and the relative smallness of the perturbations to their orbits (due to a multitude of factors such as the planets’ and the Sun’s massive size compared to almost everything else in the Solar System, the vast distances between the planets and orbital resonance), we are still able to make relatively high accuracy predictions by simplifying the Solar System into simpler models without having to solve the n-body problem using various methods of approximation such as the two-body and three-body problem[[10]](#footnote-6661). However, in the long run, these perturbations are cumulative, and may cause drastic changes and the destabilization in the planet’s orbits, which are unpredictable and are highly sensitive to the initial conditions of the Solar System[[11]](#footnote-27698).

**Chaotic behaviour caused by orbital resonance overlaps:**

Orbital resonance is defined as a configuration where the orbital periods of two objects are expressible as a simple fraction of each other[[12]](#footnote-1841). Orbital resonance is prevalent in the Solar System between different celestial bodies such as between 3 of Jupiter’s moons, Io, Ganymede and Europa in a 1:2:4 ratio[[13]](#footnote-19415) relative to Jupiter and the orbital resonance of the ratio 13:8 between Venus and Earth relative to the Sun[[14]](#footnote-20907).

A prevalent example of such perturbations is from the effect of orbital resonance overlaps between orbital resonances of different celestial bodies. While orbital resonance overlaps can be a source of stability for the planet’s orbits in our Solar System for some initial conditions, in others, they can be a source of chaos, which may significantly deviate the planet’s orbits in the long run, due to orbital resonance overlaps increasing the complexity of the gravitational interactions between celestial bodies that is sensitive to the initial conditions of the Solar System[[15]](#footnote-13031).

**Comparison between chaos in the inner planets (Mercuy, Venus, Earth, Mars) and outer planets (Jupiter, Saturn, Uranus, Neptune):**

It is found that while the inner planets of the Solar System is more predictable than the outer planets due to their smaller masses, and hence weaker orbital resonances and orbital resonance overlaps, their predictability exponentially diverges in a shorter time frame within tens of millions of years. While the outer planets, while more chaotic, generally exhibit more predictable behaviour in the long run even after tens of millions of years[[16]](#footnote-21432).

**Conclusion:**

Chaos in the Solar System is a vast topic. Apart from the chaotic nature of the planets orbits, many other aspects of the Solar System are found to exhibit chaotic behaviour such as in the asteroid belt[[17]](#footnote-27691) as well as the obliquity of the planets[[18]](#footnote-10149). Furthermore, there are also other factors not discussed in this report that can cause chaotic behaviour in the Solar System such as stellar flybys beyond the Solar System[[19]](#footnote-4840).

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